Ask Marilyn: the mathematical controversy in Parade Magazine

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Marilyn vos Savant is listed in the *Guinnes Book of World Records Hall of Fame* as the human being with the highest IQ; she makes herself useful by answering questions in her column *Ask Marilyn*, which appears, for example, in *Parade Magazine*, a publication inserted into the Sunday editions of several American newspapers. In the autumn of 1990, she solved the following problem which had been submitted for her consideration [1]:

> *Question*: Suppose you are on a game show and given a choice of three doors. Behind one is a car; behind the others are goats. You pick door No. 1, and the host, who knows what is behind them, opens No. 3, which has a goat. He then asks if you want to pick No. 2. Should you switch?
> *Answer*: Yes. The first door has a 1/3 chance of winning, but the second has a 2/3 chance.

Shortly thereafter, Savant received an avalanche of harsh rebukes from PhD's on the faculties of several American universities, reproving her for giving, as they claimed, the wrong answer; the instructors, three of whose names were published in the issue of 2 December 1990, argued that once the host had opened the losing third door, both the first and second doors then each had a probability of $\frac{1}{2}$ of winning. Savant was contumacious in her refusal to recant, even after being contradicted by such authorities, and she presented the following table in support of her solution.

DOOR 1	DOOR 2	DOOR 3
AUTO (Switch and you lose)	*GOAT*	*GOAT*
GOAT (Switch and you win)	*AUTO*	*GOAT*
GOAT (Switch and you win)	*GOAT*	*AUTO*
AUTO (Stay and you win)	*GOAT*	*GOAT*
GOAT (Stay and you lose)	*AUTO*	*GOAT*
GOAT (Stay and you lose)	*GOAT*	*AUTO*

She also suggested that the doubting Thomases use the Monte Carlo Method to discover the probabilities. Her unbridled audacity caused her to receive thousands of letters of protest; several condemnatory messages emanating from the highest officials of mathematics were quoted in the 17 February 1991 number of *Parade*. Of the innumerable PhD's censuring her, the names of twelve were published (three on 2 December 1990 and nine on 17 February 1991), and the names of five of these could be found in the *1990–1991 Combined Membership List* of the *American Mathematical Society*, the *Mathematical Association of America*, and the *Society for Industrial and Applied Mathematics*. All in all, 92% of the mail Savant received went against her, and of the letters from universities, 65% were against her. Included among her enemies were the Deputy Director of the Center for Defense Information and a research mathematical statistician from the National Institutes of Health.

If the solution to the problem is written down pompously with all high falutin formalities, then the professors who abuse Savant must surely concede the victory of their antagonist. Let C_i be the event that the car is behind the ith door; then the prior probabilities (prior to the host's intervention, that is) are clearly $\mathrm{P}(C_i) = 1/3$, $i = 1, 2, 3$, as all agree. "Without loss of generality", as they say, we may assume that the contestant chooses the first door. Note that the host is not fair; he is biased and will always open a losing door. Let R be the event that the biased host opens door No. 3. Then the laws of probability ([2], p 59) teach us that

$$\begin{aligned}\mathrm{P}(R) &= \mathrm{P}(C_1)\,\mathrm{P}(R/C_1) + \mathrm{P}(C_2)\,\mathrm{P}(R/C_2) + \mathrm{P}(C_3)\,\mathrm{P}(R/C_3)\\ &= \tfrac{1}{3}.\tfrac{1}{2} + \tfrac{1}{3}.1 + \tfrac{1}{3}.0 = \tfrac{1}{2},\end{aligned}$$

where $\mathrm{P}(A/B)$ is the conditional probability of A given B. The laws of conditional probability ([2], p 56) further assure us that

$$\mathrm{P}(C_1/R) = \frac{\mathrm{P}(R/C_1)\,\mathrm{P}(C_1)}{\mathrm{P}(R)} \quad \text{and} \quad \mathrm{P}(C_2/R) = \frac{\mathrm{P}(R/C_2)\,\mathrm{P}(C_2)}{\mathrm{P}(R)}.$$

If, now, we make the necessary substitutions into these formulas, we get

$$\mathrm{P}(C_1/R) = \frac{\tfrac{1}{2}.\tfrac{1}{3}}{\tfrac{1}{2}} = \tfrac{1}{3} \quad \text{and} \quad \mathrm{P}(C_2/R) = \frac{1.\tfrac{1}{3}}{\tfrac{1}{2}} = \tfrac{2}{3},$$

which are precisely the answers given by Savant. Her tormentors deplorably persist in solving the case when the host is fair; that is, they let G_3 be the event that there is a goat behind the third door, and they calculate

$$\mathrm{P}(C_1/G_3) = \frac{\mathrm{P}(G_3/C_1)\,\mathrm{P}(C_1)}{\mathrm{P}(G_3)} = \frac{1.\tfrac{1}{3}}{\tfrac{2}{3}} = \tfrac{1}{2}$$

and

$$\mathrm{P}(C_2/G_3) = \frac{\mathrm{P}(G_3/C_2)\,\mathrm{P}(C_2)}{\mathrm{P}(G_3)} = \frac{1.\tfrac{1}{3}}{\tfrac{2}{3}} = \tfrac{1}{2}.$$

References

1. "Ask Marilyn" by Marilyn vos Savant in *Parade Magazine*, 2 December 1990, p 25, and 17 February 1991, p 12.
2. John E. Freund, *Mathematical Statistics*, second edition, Prentice-Hall Inc., Englewood Cliffs, New Jersey, 1971.

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Superautomorphic numbers

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1. An automorphic number is generally understood to be an integer the final digits of whose square (and therefore of any integral power) form the original number, i.e. a is automorphic if $a^2 \equiv a \pmod{10^n}$, where n is the number of digits of a. There are several references to these in books on recreational mathematics, some using also bases other than 10, but otherwise generally confined to definitions and examples. Investigating some elementary properties that I had not seen treated in print led me to submit an article to the *Gazette*, which was published in October 1989 (no. 465, p 212). This caught the attention of Dr R. J. Cook, of Frome, who kindly sent me the results of some experiments it had led him to make on applying the idea to indices >2. These results were surprising and at first puzzling, but a lengthy subsequent correspondence has led to the emergence of a remarkably complicated and quite fascinating picture, which this article is an attempt to summarise. Ron Cook's computing facilities are superior to mine, and this is really the result of a joint effort in which I have supplied the theory while Ron has been responsible for most of the number-crunching and for many ideas. Although in a way the present article is a sequel to the previous one, in order that it should be self-contained there is a slight but unavoidable overlap (and anyway the treatment, being more general, is somewhat different).

2. I propose to extend the definition of an automorphic number as follows:

If m is an integer $\geqslant 2$ and a an integer of n digits such that $a^m \equiv a \pmod{10^n}$, then I call a an automorphic number (AN) of order m and length n. (Note that it will be found necessary in practice to allow a to commence with one or more zeros, e.g. 625 and 0625 are to be regarded as distinct AN's of order 2 and lengths 3, 4 respectively, because $625^2 = 390\,625$.) I denote by mAc the set of AN's of order m whose final digit is c (later it will be found convenient to allow c to be an integer >9).

3. Clearly $mA0 = \{0, 00, 000, \ldots\}$ for all m, and the possibility $c = 0$ will not be further considered. Also $mA1 \subset \{1, 01, 001, \ldots\}$ for all m; although